

CURRICULUM VITAE

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Bulgarian citizen; Canadian permanent resident

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| 20. 05. 1974 | Born in Sofia, Bulgaria; |
| 08. 1985 | Resettlement to Berlin, (former) GDR |
| 30. 06. 1990 | Abitur (school-leaving examination) at the Carl-von-Ossietzky-EOS, Berlin |
| 09. 1990 – 09. 1995 | Mathematics studies at Humboldt University Berlin |
| 10. 1995 – 05. 1998 | Promotion at Freie Universität Berlin under the supervision of Prof. Elmar Vogt. The thesis “On enumeration of chord diagrams and asymptotics of Vassiliev invariants” has been given the note <i>magna cum laude</i> . |
| 06. 1998 – 03. 1999 | Postdoc position at the Mathematics Institute of the Ludwig-Maximilians-University Munich |
| 04. 1999-02. 2003 | Visitor position at Max-Planck-Institut für Mathematik, Bonn, including |
| 01. 2001-12. 2002 | Habilitandenstipendium DFG for Habilitation at University of Bonn |
| 10. 2004-04. 2006 | JSPS Postdoc fellowship, University of Tokyo |
| 04. 2006-03. 2007 | COE Program Researcher, Research Institute for Mathematical Sciences, Kyoto University |

Conferences and talks

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| July 1995 | Ph.D. Summer School on “Geometry and Physics”, Odense, Denmark |
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April 1997	Knot theory workshop in Narbonne, France (talk given)
June 1998	Workshop "Rencontres Dijonnaises autour des nœuds et des tresses", Dijon, France
August 7-15, 1998	International Knot Theory Meeting "Knots in Hellas, 98", Delphi, Greece (talk given)
June 18-24, 1999	Arbeitstagung MPI Bonn '99
May 24-25, 2000	Clay Institute Millenium Event, Paris, France
June 5-8, 2000	Workshop "Journées Toulousaines autour des tresses et des nœuds" in Toulouse, France
June 18-24, 2000	"Perspectives of Mathematics", Goslar, Germany
March 26 - April 7, 2001	"Symplectic and Contact Topology, Quantum Cohomology, Symplectic Field Theory and Higher-Dimensional Gauge Theory", The Fields Institute, Toronto and the Centre de recherches mathématiques, Montréal
June 15-17, 2002	Canadian Mathematical Society Summer Meeting 2002, Université Laval, Quebec City, Quebec
August 12-16, 2002	Geometric Topology, Satellite Conference of ICM 2002 Beijing, Shaanxi Normal University, Xi'an, China (invited speaker, but could not attend because of lack of travel funding)
July 8-14, 2004	KOOK Seminar International for Knot Theory and Related Topics, International Conference Center in Awaji-Shima, Japan (talk 'On mutations and Vassiliev invariants (not) contained in knot polynomials')
October 25-27, 2004	Intelligence of Low Dimensional Topology 2004, Osaka City University (talk 'The second coefficient of the Jones polynomial')
December 23-26, 2004	Topology of Knots, Tokyo Woman's Christian University
February 12-14, 2005	Tohoku Knot Seminar in Akita, ALVE, Akita-shi, Japan (talk 'Brau representation and braid index')
August 1-6, 2005	The Second East Asian School of Knots and Related Topics in Geometric Topology, Dalian University of Technology, Dalian, China (talk 'The existence of achiral knot diagrams')
August 28 - September 1, 2005	"Geometry and Algebra of Knots and manifolds", Konan University, Kobe city, Japan
October 12-14, 2005	Knot Seminar (-in Zao-), Yamagata Zao, International Zaokoegen Hotel (talk 'Concordance and Thurston-Bennequin invariants of positive/negative knots and links')
December 23-26, 2005	Topology of Knots VIII, Waseda University, Tokyo (talk 'Weight systems of trivalent graphs and hyperbolic volume of alternating knots by genus')
February 17-20, 2006	Hiroshima Topology Conference (Celebrating Prof. Takao Matumoto's 60th birthday), Hiroshima University, Higashi-Hiroshima city, Japan
July 22-27, 2006	Intelligence of Low Dimensional Topology 2006, Hiroshima University,

Higashi-Hiroshima city, Japan (talk 'Mutation and the colored Jones polynomial')

Further talks (not attached to conferences)

"Some applications of link polynomial evaluations" (Vortrag am Graduiertenkolleg University Bonn, 11/6/1999)

"Wheel graphs, Lucas numbers and the determinant of a knot" (MPI-Oberseminar, 30/3/2000)

"On the number of links and link polynomials" (Oberseminar Topologie, MPI, 11/12/2000)

"Canonical genus and the signature" (Kobe Topology Seminar, Kobe University, November 24, 2001)

"Special diagrams and the positivity of the signature" (Nara Topology Seminar, Nara Women's University, November 29, 2001)

"On the signature of positive links" (Mathematics Institute Seminar, University Bonn, 19/4/2002, and Oberseminar Topologie, MPI, 13/1/2003, Topology Seminar Université Lille 14/11/2003)

"On some relations between hyperbolic volume and combinatorial knot invariants" (Topology seminar, University of Toronto, 26/3/2003)

"Square Fibonacci numbers and linear recurrence sequences" (Graduate seminar, University of Toronto, 17/4/2003)

"Four Color Theorem, Lie Algebras, hyperbolic volume and enumeration of alternating knots by genus" (Topology Seminar, Tokyo Institute of Technology, 3/3/2004)

"On mutations and Vassiliev invariants (not) contained in knot polynomials" (Friday seminar on Knot Theory, Osaka City University, 11/6/2004)

"Properties of closed 3-braids" (Topology Seminar, University of Tokyo, 30/11/2004)

"Properties of closed 3-braids" (KOOK Seminar, Osaka City University, 2/4/2005)

"Applications of braiding sequences" (Research Institute for Mathematical Sciences, Topology Seminar, 17/11/2005)

"Braiding sequences and Thurston-Bennequin invariants" (Hokkaido University, Department of Mathematics Colloquium 21/12/2005)

"Knots and their crossing numbers" (Research Institute for Mathematical Sciences, Colloquium, 10/5/2006)

"Estimation of crossing numbers of knots" (Research Institute for Mathematical Sciences, Topology Seminar, 13/7/2006)

Lectures (incl. series of talks and minicourses)

"Non-associative tangles" (following a paper of D. Bar-Natan, "Geometric topology, Proceedings of the Georgia international topology conference", W. H. Kazez, ed., 139-183, Amer. Math. Soc. and International Press, Providence, 1997), Lecture series, knot theory seminar, Humboldt University Berlin, 1993/1994.

"Algebraic structures on modules of diagrams" (following P. Vogel, announced to appear in Invent. Math. since about 1998), Lecture series, knot theory seminar, Humboldt University Berlin, 1994/1995.

"Some applications of braiding sequences", series of 4 talks: "A bound for the number of restricted Vassiliev invariants", "The canonical genus" (2 talks), "On the crossing number of semiadequate links", Low dimensional topology Seminar, Research Institute for Mathematical Sciences, Kyoto University, November 16, 17, 30, 2001.

"Mathematical English" (rules of writing mathematical papers in English), Osaka City University, June 10, 17, 24, July 1, 2004

Invitations and Working visits

April 1996	Strasbourg (France), C. Kassel
June 1996	Toulouse (France), T. Fiedler
March 1999	Toulouse, T. Fiedler
June 1999	Zurich, V. Chernov
November 1999	Institut des Hautes Études Scientifiques in Bures-sur-Yvette (France)
January-July 2001	University of Toronto, Kunio Murasugi
November 2001	Research Institute for Mathematical Sciences, Kyoto University (Japan), Hitoshi Murakami
May 2003	Toulouse, T. Fiedler
October 2003	Waterloo (Canada), L. Kauffman
November 2003	Lille (France), S. Kallel; Toulouse, T. Fiedler
February-March 2004	Tokyo Institute of Technology, Hitoshi Murakami
June-July 2004	Osaka City University, Akio Kawauchi

Administrative matters

Organizer: Graduate Student Seminar, Mathematics Institute, Ludwig-Maximilians-University Munich, 06. 1998 – 03. 1999

Session Chairman:

International Graduate Course Student Workshop for Knot Theory and Related Topics, Osaka City University, Media Center, July 5-7, 2004;

Tohoku Knot Seminar in Akita, ALVE, Akita-shi, Japan, February 12-14, 2005.

Refereeing

Journals: Topology, Compositio Mathematica, Documenta Mathematica, J. of Knot Theory and Its Ramifications.

Organizations and Services: AMS Math Reviews, Zentralblatt, National Science Foundation

Publications

(selected and ordered approximately chronologically by time of writing, not of publication)

- [BS] *The Fundamental Theorem of Vassiliev invariants*, joint with D. Bar-Natan, "Geometry and Physics", Lecture Notes in Pure & Appl. Math. **184**, M. Dekker, New York, 1996, 101–134.
- [St] *Über Harrison-Kohomologie und die Drinfel'd-Vermutung*, diploma thesis, Humboldt University, Berlin, 1995
- [St2] *Enumeration of chord diagrams and an upper bound for Vassiliev invariants*, J. Of Knot Theory and Its Ram. **7(1)** (1998), 93–114.
- [St3] *Stirling numbers, Eulerian idempotents and a diagram complex*, J. Of Knot Theory and Its Ram. **7(2)** (1998), 231–256.
- [St4] *A Survey on Vassiliev Invariants for knots*, "Mathematics and Education in Mathematics", Proceedings of the XXVII. Spring Conference of the Union of Bulgarian Mathematicians, 1998, 37–47.
- [St5] *On enumeration of chord diagrams and asymptotics of Vassiliev invariants*, Doctor thesis, Freie University Berlin, 1998.
- [St6] *Gauß sum invariants, Vassiliev invariants and braiding sequences*, J. Of Knot Theory and Its Ram. **9(2)** (2000), 221–269.
- [St7] *On finiteness of Vassiliev invariants and a proof of the Lin-Wang conjecture via braiding polynomials*, J. Of Knot Theory and Its Ram. **10(5)** (2001), special volume for the proceedings of the International Conference on Knot Theory "Knots in Hellas, 98", 769–780.
- [St8] *Vassiliev invariants on fibered and mutually obverse knots*, J. Of Knot Theory and Its Ram. **8(4)** (1999), 511–519.
- [St9] *The braid index and the growth of Vassiliev invariants*, J. Of Knot Theory and Its Ram. **8(6)** (1999), 799–813.
- [St10] *On the number of chord diagrams*, Discr. Math. **218** (2000), 209–233.
- [St11] *Genera of knots and Vassiliev invariants*, J. Of Knot Theory and Its Ram. **8(2)** (1999), 253–259.
- [St12] *On some restrictions to the values of the Jones polynomial*, Indiana Univ. Math. J. **54 (2)** (2005), 557–574.
- [St13] *Positive knots, closed braids and the Jones polynomial*, math/9805078, Ann. Scuola Norm. Sup. Pisa Cl. Sci. **2(2)** (2003), 237–285.
- [St14] *Some minimal degree Vassiliev invariants not realizable by the HOMFLY and Kauffman polynomial*, C. R. Acad. Bulgare Sci. **54(4)** (2001), 9–14.
- [St15] *Mutant links distinguished by degree 3 Gauss sums*, Proceedings of the International Conference on Knot Theory "Knots in Hellas, 98", Series on Knots and Everything **24**, World Scientific, 2000.
- [FS] *New knot and link invariants*, joint with T. Fiedler, Proceedings of the International Conference on Knot Theory "Knots in Hellas, 98", Series on Knots and Everything **24**, World Scientific, 2000.
- [St16] *Gauss sums on almost positive knots*, Compositio Mathematica **140(1)** (2004), 228–254.
- [St17] *The granny and the square tangle and the unknotting number*, Topol. Appl. **117** (2002), 59–75.
- [St18] *Knots of genus one*, Proc. Amer. Math. Soc. **129(7)** (2001), 2141–2156.
- [St19] *The Conway Vassiliev invariants on twist knots*, Kobe J. Math. **16(2)** (1999), 189–193.
- [St20] *Vassiliev invariants and rational knots of unknotting number one*, math/9909050, Topology **42(1)** (2003), 227–241.

- [St21] *The crossing number and maximal bridge length of a knot diagram*, with an appendix by M. Kidwell, Pacific J. Math. **210(1)** (2003), 189–199.
- [St22] *Polynomial values, the linking form and unknotting numbers*, math.GT/0405076, Math. Res. Lett. **11(5-6)** (2004), 755–769.
- [St23] *Square numbers, spanning trees and invariants of achiral knots*, math.GT/0003172, Comm. Anal. Geom. **13(3)** (2005), 591–631.
- [St24] *The Jones polynomial, genus and weak genus of a knot*, Ann. Fac. Sci. Toulouse **VIII(4)** (1999), 677–693.
- [St25] *On Unknotting Numbers and Knot Trivadjency*, On unknotting numbers and knot trivadjency. Math. Scand. **94(2)** (2004), 227–248.
- [St26] *On the unknotting number of minimal diagrams*, Mathematics of Computation **72(244)** (2003), 2043–2057.
- [St27] *Branched cover homology and Q evaluations*, Osaka J. Math. **39(1)** (2002), 13–21.
- [St28] *Rational knots and a theorem of Kanenobu*, Exper. Math. **9(3)** (2000), 473–478.
- [St29] *Fibonacci numbers and the ‘fibered’ Bleiler conjecture*, Int. Math. Res. Notices **23** (2000), 1207–1212.
- [St30] *The signature of 2-almost positive knots*, J. Of Knot Theory and Its Ram. **9(6)** (2000), 813–845.
- [St31] *Some examples related to 4-genera, unknotting numbers, and knot polynomials*, Jour. London Math. Soc. **63(2)** (2001), 487–500.
- [St32] *On the coefficients of the link polynomials*, Manuscr. Math. **110(2)** (2003), 203–236.
- [St33] *Some inequalities between knot invariants*, Internat. J. Math. **13(4)** (2002), 373–393.
- [St34] *On the crossing number of positive knots and braids and braid index criteria of Jones and Morton-Williams-Franks*, Trans. Amer. Math. Soc. **354(10)** (2002), 3927–3954.
- [St35] *Some applications of Tristram-Levine signatures*, Adv. Math. **194(2)** (2005), 463–484.
- [KS] *Examples Relating to the Crossing Number, Writhe, and Maximal Bridge Length of Knot Diagrams*, joint with M. Kidwell, Mich. Math. J. **51(1)** (2003), 3–12.
- [STV] *The canonical genus of a classical and virtual knot*, joint with V. Tchernov and A. Vdovina, Geometriae Dedicata **95(1)** (2002), 215–225.
- [St36] *On the number of links and link polynomials*, Quart. J. Math. Oxford **55(1)** (2004), 87–98.
- [St37] *The skein polynomial of closed 3-braids*, J. Reine Angew. Math. **564** (2003), 167–180.
- [HS] *Examples of knots without minimal string Bennequin surfaces*, joint with M. Hirasawa, Asian Journal Math. **7(3)** (2003), 435–446.
- [St38] *On the Polyak-Viro Vassiliev invariant of degree 4*, to appear in Canad. Math. Bull.
- [SV] *Counting alternating knots by genus*, joint with A. Vdovina, Math. Ann. **333** (2005), 1–27.
- [St39] *On polynomials and surfaces of variously positive links*, Jour. Europ. Math. Soc. **7(4)** (2005), 477–509.
- [MS] *The Alexander polynomial of planar even valence graphs*, joint with K. Murasugi, Adv. Appl. Math. **31(2)** (2003), 440–462.
- [St40] *Newton-like polynomials of links*, Enseign. Math. (2) **51(3-4)** (2005), 211–230.
- [St41] *On cabled knots and Vassiliev invariants (not) contained in knot polynomials*, to appear in Canad. J. Math.
- [SSW] *Euclidean Mahler measure and twisted links*, joint with D. S. Silver and S. G. Williams, Algebr. Geom. Topol. **6** (2006), 581–602.
- [St42] *Hard to identify (non-)mutations*, to appear in Math. Proc. Cambridge Philos. Soc.

Partial contributions to the following papers and monographs:

- [Fi] Th. Fiedler, *Gauss Diagram Invariants for Knots and Links*, Kluwer Academic Publishers, Mathematics and Its Applications Vol **532** (2001).
- [Fi2] ———, *Gauss diagram invariants for knots which are not closed braids*, Math. Proc. Cambridge Philos. Soc. **135(2)** (2003), 335–348.

- [Mo] H. R. Morton (ed.), *Problems*, Ser. Knots Everything **24** (Knots in Hellas '98, Delphi), World Sci. Publishing 2000, 547–559.
- [Oh] T. Ohtsuki (ed.), *Problems on invariants of knots and 3-manifolds*, Geometry and Topology Monographs **4** (2002) (Invariants of knots and 3-manifolds, Kyoto 2001), 377–572.
- [Za] D. Zagier, *Vassiliev invariants and a strange identity related to the Dedekind eta-function*, *Topology* **40(5)** (2001), 945–960.

References (some mathematicians who can be consulted about me and/or my work)

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1 Subject and Achievement of past Research

My research covers several areas of knot theory, with relations to combinatorics, number theory and algebra, which are outlined below (and by no means unrelated).

1.1 Vassiliev invariants

My first work was to improve the upper bound on the dimension of Vassiliev invariants of degree D . The best known previous upper bound was $(D-2)!/2$ due to Ng. It was known that Vassiliev invariants can be understood combinatorially in terms of chord diagrams modulo the $4T$ relation. I introduced a certain type of chord diagrams and showed that they generate modulo $4T$ relations the space of Vassiliev invariants. Then I estimated from above the number of such chord diagrams to $D!/1.1^D$. Later, Zagier showed that the generating series of the numbers of such chord diagrams occurs in a strange identity related to the Dedekind eta-function. He found the exact asymptotical behaviour of these numbers, improving the number 1.1 to $\pi^2/6$, thus establishing the currently best upper bound.

My later work were constructions of knots with Vassiliev invariants of bounded degree and specific properties, like given unknotting numbers, signatures and 4-genera. I showed the non-existence of Vassiliev invariants that depend on any finite number of link polynomial coefficients (except the Conway/Alexander polynomial).

1.2 Legendrian knots

Legendrian knots are called knots embedded in the standard contact space. There are inequalities relating the Thurston-Bennequin invariant and Maslov number of Legendrian knots and the degrees of the polynomial invariants of the underlying topological knots. Using these inequalities I gave estimates of the invariants of Legendrian negative knots. This result can be considered as a generalization of Kanda's determination of the maximal Thurston-Bennequin invariant of the negative trefoil.

1.3 Gauss diagram formulas

Fiedler and Polyak-Viro introduced a new approach to defining Vassiliev invariants by explicit formulas. Such formulas involve sums over specific tuples of crossings of a knot or link diagram of functions involving writhes of the crossings (similarly to linking numbers). These formulas proved useful in the study of positive knots (knots with diagrams all of whose crossings are positive). Positive knots and links have been studied, beside because of their intrinsic knot-theoretical interest, with different motivations and in a variety of contexts, including singularity theory, algebraic curves, dynamical systems, and (in some vague and yet-to-be understood way) in 4-dimensional QFTs. Using the Fiedler-Polyak-Viro formulas, I found several inequalities between Vassiliev invariants of positive knots, allowing to exclude certain knots from having this property. Later I sought generalizations of some criteria to almost positive knots.

1.4 Canonical Seifert surfaces

The set of knot diagrams whose canonical Seifert surfaces (that is, surfaces obtained by Seifert's algorithm) of given (canonical) genus admits a structure of generating series. It allows to prove,

for example, that the number of alternating knots of fixed genus grows polynomially in the crossing number.

1.5 Non-trivial Jones polynomial problem

In 1985 Jones discovered the famous polynomial invariant named after him and asked if it distinguishes all non-trivial knots from the trivial one. His question remains unanswered despite the recent solution for links. I showed that semiadequate links, as defined by Lickorish-Thistlethwaite, have non-trivial Jones polynomial. Montesinos links are semiadequate, and then I showed that so are 3-braid links, so the non-triviality result applies to these classes.

1.6 Closed 3-braids

I classified among closed 3-braid links the braid positive, strongly quasipositive and fibered ones. Then I showed that 3-braid links with given (non-zero) Alexander or Jones polynomial are finitely many, and can be effectively determined. In recent joint work with M. Hirasawa and M. Ishiwata we showed that 3-braid links have a unique incompressible Seifert surface.

1.7 Knot tables

For some time I have been interested in using knot tables, compiled by Hoste, Thistlethwaite, and Weeks, to seek knots with interesting properties, and so to provide examples and counterexamples to problems that have remained inaccessible using (entirely) manual reasoning.

1.8 Other topics

I have also done some work on unknotting numbers, link polynomials, number theoretic properties of knot invariants, and enumeration problems of links.

2 Present and future Research

I will list now some main topics of research I'm currently interested in, and plan to work on in the visible future. My choice of problems is not fixed, and will also depend on my interaction with other mathematicians I expect to meet. After every topic I will also briefly explain the expected results of its investigation.

2.1 Hyperbolic volume

There have been so far several situations, in which the hyperbolic volume exhibits a relation to a combinatorially defined knot invariant. The most important one is Kashaev's conjecture on values of colored Jones polynomials, popularized by H. Murakami.

Another correspondence was observed by Brittenham, namely that the volume is bounded on alternating knots of given genus. His bound can be improved by inequalities of Lackenby-Agol-Thurston involving an invariant of knot diagrams called twist number. These bounds are also

related to conjectures of Dunfield, namely that the volume linearly approximates a logarithm of the determinant of alternating knots. I'm currently interested in obtaining and improving such inequalities.

2.2 Trivalent graphs and enumeration of knots by genus

There is also a relation between the Brittenham approach and the enumeration problems of knots of given genus, and the sl_N weight system of trivalent graphs known from the theory of Vassiliev invariants.

One can express the maximal volume of knots of given canonical genus by links L_G associated to planar trivalent graphs G similarly to Habiro's claspers. The sl_N weight system of G then is related to both the hyperbolic volume and the enumeration of knots by genus, and also to the enumeration of 1-vertex triangulations of oriented surfaces carried out by my collaborator A. Vdovina.

2.3 Weight system-volume-conjectures

The form of the relation between the sl_N weight system W_N of G and volume of L_G is not yet clear, but calculations suggest that definitely something is going on beyond accidental coincidences. I hope in the future to deepen my understanding of hyperbolic volumes, in particular to understand better these relations. I hope also to find out whether and what is a relation of these coincidences to the Volume conjecture. For example, can one calculate the colored Jones polynomials of L_G , and establish the relation modulo the Volume conjecture? Can one gain insight into the Volume conjecture from these relations? This is also linked to understanding the, in particular multiplicative, structure of the sl_N weight systems of G . Few facts are known, including the multiplicative character of Vogel's algebra and Bar-Natan's version of the 4-Color-Theorem. A new observation from the work of Bacher and Vdovina on 1-vertex triangulations is that the linear term of the sl_N weight systems vanishes in Euler characteristic < -1 . Their work also implies bounds on the number of linear monomials in the calculation of W_N , that in turn bound the asymptotical growth of the number of alternating knots of given genus, which I seek to improve. I hope to progress on at least some of these many interrelated problems in the future.

2.4 Gauss sum invariants

There are still opportunities left in the application of Gauss sum invariants to positive knots and related knot classes, most naturally, to improve the existing inequalities. More importantly, a computational project is to implement Fiedler's new character Gauss sum invariants (which take as input not a single diagram, but a sequence of diagrams of knots in the solid torus), in the hope to distinguish knot orientation with them, after the success on braids. This will obviously give a huge impetus on the theory of Gauss sum invariants.

2.5 Number theoretic properties of knot invariants

One of my original mathematical interests was number theory (my specialization turned into a different direction by the influence I experienced during my studies). I'm interested in situations in which knot invariants can be studied from the point of view of some elementary number theoretic properties. A series of problems I intend to work on is related to determinants of achiral knots with particular properties, for example unknotting number one. These determinants are sums of two

squares. It would be interesting to study which such numbers occur in which situations. Number theoretic properties of the determinant have also applications to unknotting numbers and knot distance, and maybe I can find more such applications.

2.6 Non-trivial Jones polynomial problem

I seek further generalizations of the non-triviality result for the Jones polynomial, for example to arborescent knots. I also try to prove that there are infinitely many positive knots with no positive minimal crossing diagrams (a problem raised by Nakamura), and achiral knots of any odd crossing number at least 15.

2.7 Mutation and the colored Jones polynomial

In joint work with Toshifumi Tanaka, we found examples of knots with the same polynomial invariants and hyperbolic volume, with variously coinciding 2-cable polynomials and colored Jones polynomials, which are not mutants. In particular, we show that there exists an infinite family of pairs of hyperbolic knots with equal colored Jones polynomial, which are not mutants. This answers a question of Przytycki.

2.8 5-moves and Montesinos links

Extending work of Ishiwata, I determined the 5-move equivalence classes of Montesinos links up to mutation; one obtains from this a Jones and Kauffman polynomial test for a Montesinos link. To remove 'up to mutation', I must determine what of the Montesinos links $(1/2, \dots, 1/2, 2/5, \dots, 2/5)$ and their mutants (permute the $1/2, 2/5$ s) are 5-move equivalent. A 5-move preserves the 5-Burnside group of the link group, and maps 1-to-1 the homomorphisms to a fixed 5-group, so I explore group theoretic ways to accomplish this.

2.9 Other topics

I'm also working on some problems of braids, for example the question of Rudolph whether (strongly) quasipositive knots have (strongly) quasipositive braid representations of minimal strand number. I expect counterexamples but they are not easy to construct. (I had in the past some counterexamples for positive braid representations and, jointly with M. Hirasawa, minimal genus band representations.)